Available online at www.sciencedirect.com
SCIENCE DIRECT

Journal of Sound and Vibration 263 (2003) 917–935

JOURNAL OF SOUND AND VIBRATION

www.elsevier.com/locate/jsvi

A parametric study on fluid–structure interaction problems

Damodar Maity[a,*], Sriman Kumar Bhattacharyya[b]

[a] *Department of Civil Engineering, Indian Institute of Technology, Guwahati 781 039, Assam, India*
[b] *Department of Civil Engineering, Indian Institute of Technology, Kharagpur 721 302, West Bengal, India*

Received 18 December 2001; accepted 17 July 2002

Abstract

This paper deals with finite element analysis of the fluid–structure systems considering the coupled effect of elastic structure and fluid. The equations of motion of the fluid considered inviscid and compressible are expressed in terms of the pressure variable alone. The elastic structure and the fluid domain are treated as two separate systems and discretized with finite elements. The solution of the coupled system is accomplished by solving the two systems separately with the interaction effects at the fluid–solid interface enforced by a developed iterative scheme. Non-divergent pressure and displacement are obtained simultaneously through a few numbers of iterations. Studies show the accuracy of the proposed algorithm, while comparing with the existing ones available in the literature. The parametric study of the coupled system shows the importance of fluid height and material property of the structure.

1. Introduction

The dynamic interaction between an elastic structure and a compressible fluid has been the subject of intensive investigations in recent years. It is well known that the problem in question can be formulated at different levels of complexity and 'completeness' of physical representatively. From the classical Westergaard, or 'added mass', approach in which water incompressibility and rigid structure are assumed, one may introduce, separately or in connection, the compressibility of water and the flexibility of structure. Some simplified approaches are available in which fluid–structure interaction is studied in a decoupled manner. In this type of analysis, the fluid response is first obtained assuming the structure to be rigid and the resulting pressure field is imposed on the structure to obtain the structural response. Though such type of analysis leads to a conservative

*Corresponding author. Tel.: +91-361-690-321 to 28x2113; fax: +91-361-690-762.
E-mail addresses: damodar@iitg.ernet.in, damodarmaity@yahoo.com (D. Maity), bsri@civil.iitkgp.ernet.in (S.K. Bhattacharyya).

0022-460X/03/$ - see front matter
PII: S0022-460X(02)01079-9

design of the structure, if resonance between the structure and the energy release mechanism in the fluid occur it can lead to the development of unsound design. Moreover, if coupled modes are excited, this approach gives non-conservative results. Thus it is necessary to study the fluid–structure interaction problems in a coupled manner considering the flexibility effect of the structure. The most common approach being adopted at present is that both the systems are coupled and solved as one system [1–4]. Formulations based on displacement variables are generally chosen for the structure while the fluid is described by different variables such as displacement, pressure, velocity potential, etc. for such coupled problems. A number of researchers [4–6] used hydrodynamic pressure as the unknown variable in finite element discretization of the fluid domain. But the resulting equations in this case lead to unsymmetrical matrices and require a special purpose computer program [7,8]. Zienkiewicz et al. [9] represented the equations of fluid domain in terms of a displacement potential. The coupled equations of motion in this case become unsymmetrical, but irrotationality condition on fluid motion is automatically satisfied. Many researchers [1,10–12] formulated the governing equations of fluid in terms of displacements. The advantage of the displacement-based formulation is that the fluid elements can easily be coupled to the structural elements using standard finite element assembly procedures. But the degrees of freedom for the fluid domain increase significantly (especially for 3-D problems). Moreover, the fluid displacements must satisfy the irrotationality condition, otherwise zero-frequency spurious modes may occur. The variables such as velocity and pressure have also been used for representing the governing equations for fluid by Fenves et al. [2]. However, requirement of computational time becomes higher as number of unknown parameters increase in the fluid domain. Thus the need of a large computer storage and expense of vast computer time usually make the analysis impractical. The solution of the coupled system may be accomplished by solving the two systems separately with the interaction effects enforced by iteration [13–17]. The major advantage of this method is that the coupled field problems may be tackled in a sequential manner. The analysis is carried out for each field and interaction effect is accommodated by updating the variables of the fields in the respective coupling terms.

Due to complex topographical condition of the dam structures, the finite element method is recognized as one of the powerful numerical tools in most practical problems. In the finite element analysis of such problems, difficulties arise mainly because of the large extent of the fluid domain, where fluid is practically unbounded. Hence, it is necessary to arbitrarily truncate the reservoir region in order to have a manageable computational domain. A number of far-boundary conditions have been reported in the literature and they may be broadly categorized as: (i) imposition of a boundary condition along the truncation surface [5,18–20], and (ii) coupling the finite element discretization with other type of discretization such as 'infinite elements' [21] 'boundary elements' [22,23] or with 'continuum' solutions [24,25]. There is a common belief that boundary element method is superior over finite element for the modelling of infinite or semi-infinite domains. However, in the reported literature [26], the efficiency of boundary element method in time domain analysis is not ascertained. This is because of the presence of the convolution integral and singularity of the kernels of the formulation, which requires large storage space and computational time for the evaluation of the effect of past time history and numerical integration of the kernels. Moreover, use of boundary element method requires the solution of an unsymmetrical and unbounded matrix. Hence, this method does not possess any significant advantage over the finite element method. On the other hand, the finite element approach has the distinct advantage of being straightforward in implementation. The most commonly used boundary condition along the truncation surface is the Sommerfeld radiation condition [5]. But the behaviour of the fluid motion at the truncation boundary is not represented truly. Hence, a large extent of fluid domain is required to be included in the analysis. Saini et al. [21] developed an infinite element technique for the far field, Clough et al. [27] used the finite element method, Hanna and Humar [22] used the boundary element methods. In all the above investigations, frequency-domain analyses have been used. Since as in the frequency domain, using a transmitting boundary for the reservoir does not have an exact counterpart in the time domain [14,9], seldom has time-domain analysis been employed. A fundamental assumption in all these studies is to treat the problem as two dimensional. This assumption is reasonable for gravity dams. The important conclusions from these literatures are that the compressibility of water cannot be neglected in analysis, and the elastic properties of dam may alter the pressure significantly.

To compensate the inadequacies of these analyses, a robust and efficient finite element iterative scheme is developed to study the fluid–structure interaction problems. In this paper, a time-domain analysis, modelling the infinite fluid domain into a finite one with an efficient truncation boundary condition adopted by Maity and Bhattacharyya [18], which includes the radiation effects and can be adopted in the finite element formulation in a simple form, is presented. An efficient iterative scheme, which enables to obtain simultaneously a divergent free displacement and pressure in the structure and fluid domain, respectively, at any instant of time, is developed. Since the two systems are dealt with separately, the resulting matrices are also symmetric. The pressure is considered as unknown variable parameter in the fluid domain, assuming the fluid as inviscid and irrotational. The elastic structure is analyzed by two-dimensional plane strain formulation. The response of the coupled system is obtained by solving the two systems separately with the interaction effects enforced by iteration. Parametric study has been carried out to investigate the importance of material property of the structure and the fluid height.

2. Theoretical formulation

2.1. Governing equations for fluid and boundary conditions

Neglecting the internal viscosity, and assuming the water to be linearly compressible with a small amplitude irrotational two-dimensional movement, the hydrodynamic pressure distribution in the reservoir system is governed by the pressure wave equation

$$\nabla^2 p(x, y, t) = \frac{1}{C^2}\ddot{p}(x, y, t), \tag{1}$$

where $p(x, y, t)$ is the hydrodynamic pressure distribution in excess of the hydrostatic pressure, C is the acoustic wave velocity in water, t is the time variable and x, y are the space variables. The hydrodynamic pressure distributions within the domain may be obtained by solving Eq. (1) with the following boundary conditions.

(a) *At the free surface*: Considering the effects of surface waves of the fluid, the boundary condition of the free surface is taken as

$$\frac{1}{g}\ddot{p} + \frac{\partial p}{\partial y} = 0. \tag{2}$$

(b) *At the fluid–structure interface*: Considering the structure to vibrate with an acceleration of $ae^{i\omega t}$ in which, ω is the circular frequency of vibration; and $i = \sqrt{-1}$, the condition along the fluid–structure interface can be specified as

$$\frac{\partial p}{\partial n}(0, y, t) = -\rho_f a e^{i\omega t}, \tag{3}$$

where ρ_f is the mass density of fluid, n is the outwardly directed normal to the elemental surface along the interface, and a is the acceleration of the fluid–solid interface in the direction of n.

(c) *At the fluid–reservoir bed interface*: Assuming reservoir floor to be rigid, the condition adopted is

$$\frac{\partial p}{\partial n}(x, 0, t) = 0. \tag{4}$$

(d) *At the truncation surface*: The specification of the far-boundary condition at the truncation surface has been presented elaborately in Maity and Bhattacharyya [18]. However, the details of the derived far-boundary condition can be found in Appendix A. The far-boundary condition adopted in the present case is as follows:

$$\frac{\partial p}{\partial n} = \frac{\partial p}{\partial x} = -\frac{p}{H}\zeta, \tag{5}$$

where p is the pressure of the fluid domain and ζ is given by

$$\zeta = -\frac{\sum_{m=1}^{\infty} \frac{(-1)^{m+1}}{2m-1} e^{(-f_m x/H)} \cos\left(\lambda_m \frac{y}{H}\right)}{\sum_{m=1}^{\infty} \frac{(-1)^{m+1}}{(2m-1)f_m} e^{(-f_m x/H)} \cos\left(\lambda_m \frac{y}{H}\right)}. \tag{6}$$

To get the effect of unbounded fluid domain in the truncation surface, ζ is determined numerically assuming m to be a large number.

2.2. Finite element implementation

Here the fluid domain is discretized as an assemblage of finite elements, assuming pressure to be the nodal unknown. By the use of the Galerkin process, the discretized form of Eq. (1) is obtained as

$$[E]\{\ddot{p}\} + [A]\{\dot{p}\} + \left([G] + \frac{\zeta C}{H}[A]\right)\{p\} = -\rho_f[S]\{a\}, \tag{7}$$

where $\{p\}$ represents the vector of nodal pressures for the fluid domain. Expressions for the matrices $[E]$, $[A]$ $[G]$ and $[S]$ may be found in the paper of Zienkiwicz and Newton [5]. It is important to note that owing to the implementation of the proposed far-boundary condition, the form of the discretized equation remains unchanged and there is no extra computation required except for the modification of a few elements of the matrix $[G]$. For any prescribed acceleration at the fluid–structure interface, Eq. (7) may be used to solve the hydrodynamic pressure in the fluid domain.

2.3. Governing equations for structure

The equation of motion of a structure subjected to external forces can be written in standard finite element form as

$$[M]\{\ddot{U}\} + [C]\{\dot{U}\} + [K]\{U\} = -[M]\{R\}\ddot{U}_g + \{F_e\} + \{F_h\}, \tag{8}$$

where $\{U\}$, $\{\dot{U}\}$, $\{\ddot{U}\}$ are the vector of nodal displacement, velocity and acceleration, respectively. $\ddot{U}_g$ is the ground acceleration and $\{R\}$ is the acceleration transformation matrix. Analyzing the structural system by plane strain formulation, the elementary stiffness matrices will be

$$[K^e] = \int_\Omega [B^T][D][B]\,d\Omega, \tag{9}$$

where $[D]$ is the constitutive matrix and $[\bar{B}]$ is the strain–displacement matrix at any node i. For an element, the $[\bar{B}]$ matrix can be defined by augmenting contribution from all the nodes of the element. $[M]$ is the consistent mass matrix. The elementary mass matrix with density ρ_s is

$$[M^e] = \int_\Omega [N]^T \rho_s [N]\,d\Omega. \tag{10}$$

The structural damping matrix is constructed by

$$[C] = a'[M] + b'[K]. \tag{11}$$

The constants a' and b' are chosen to control the damping proportionately. $\{F_e\}$ is the time-dependent external force on the structure, and $\{F_h\}$ is the hydrodynamic force on the structure resulting from adjacent fluid.

In the present study, eight-node quadrilateral elements for displacement and four-node linear element for pressure have been adopted for the solution of Eqs. (7) and (8) with the prescribed boundary conditions.

3. Iterative scheme

An iterative scheme has been developed to achieve the coupled effect of fluid–structure system (Fig. 1). At any instant of time t, the resulting hydrodynamic pressure is evaluated by solving the fluid domain using Eq. (7) with appropriate boundary conditions. At the same time instant, the developed pressures exert forces on the adjacent structure as hydrodynamic forces and these are functions of generated pressure. At the same instant of time, the structural element is analyzed with the forces, developed due to hydrodynamic pressures, using Eq. (8). Due to the additional forces, the structure undergoes a displacement $\{U\}_t$. As a result the fluid–solid interface boundary changes and hence the solution of the fluid domain. The fluid domain is solved again at the same time instant with the changed conditions of displaced structural boundary. Consequently, the structural system is also analyzed with the changed forces. Thus at time t, both the hydrodynamic pressure $\{p\}_t$ and the structural displacement $\{U\}_t$ are iterated simultaneously till a desired level of convergence is achieved. Thus,

$$\left|\frac{\{p_{i+1}\}_t - \{p_i\}_t}{\{p_i\}_t}\right| \leqslant \varepsilon \quad \text{and} \quad \left|\frac{\{U_{i+1}\}_t - \{U_i\}_t}{\{U_i\}_t}\right| \leqslant \varepsilon, \tag{12}$$

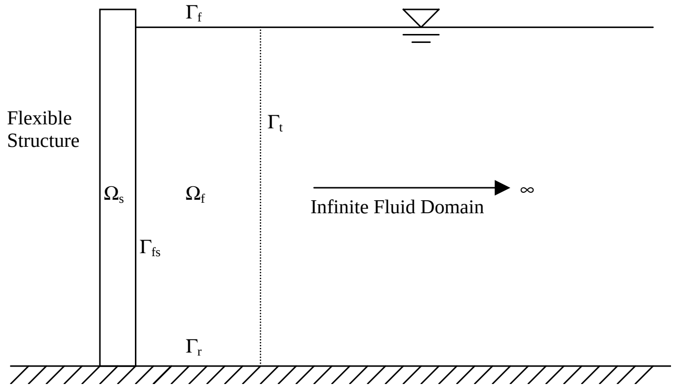

Fig. 1. Typical fluid–structure system.

where i being the number of iteration, ε is a small preassigned tolerance value. A flow chart indicating the iterative scheme for the solution scheme of coupled fluid–structure interaction problem is presented in Fig. 2. The most costly operation involved in the above algorithm is to successively solve two linear equation systems at each iteration. But in the present case, matrices involved in the solution of the system equations are decomposed into triangular forms at the beginning of the of the iteration, and thereby only two forward-eliminations and back-substitutions are required at each iteration step. Thus, the time required to obtain the coupled response for a particular time instant is minimized in the developed iterative scheme.

3.1. Computation of velocity of fluid

After computing the developed hydrodynamic pressure inside the fluid domain, the acceleration of the fluid particles is calculated from the following equation:

$$p_{,i} + \rho_f \dot{v}_i = 0, \tag{13}$$

where ρ_f is the mass density of fluid and $\dot{v}_i$ is the acceleration of the fluid particle. The velocity of the fluid particle may be evaluated from the known values of acceleration at any instant of time using well-known Gill's time integration scheme, which seems to be stable and requires less computational time over other schemes. At any instant of time t, velocity will be

$$v_t = v_{t-\Delta t} + \Delta t \dot{v}_t. \tag{14}$$

Based on the velocities computed at the Gauss points of each individual element, velocity vectors in the fluid domain are plotted.

4. Numerical results

The examples are related to the analysis of hydrodynamic pressure distribution on two-dimensional structures exposed to infinite fluid media. In the present examples, the coefficients ζ in Eq. (6) are obtained assuming m as 50, where the series converges sufficiently.

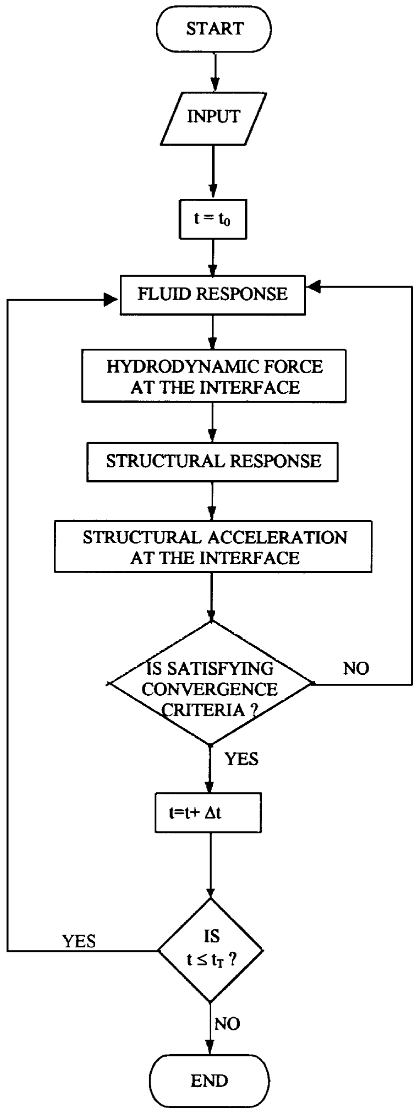

Fig. 2. Flow chart of the iterative scheme.

Example 1. *Validation of the proposed algorithm.* In order to examine the feasibility and the accuracy of the proposed iterative scheme, a benchmark problem has been solved and compared with the existing literature. The data assumed for the problem are: depth of the fluid domain $(H_f) = 180\,\text{m}$, acoustic wave speed in water $(C) = 1438.7\,\text{m/s}$, mass density of water $(\rho_f) = 1000\,\text{kg/m}^3$, modulus of elasticity of the structure $(E_s) = 3.5 \times 10^{10}\,\text{kg/m}^2$, the Poison ratio $(v) = 0.2$, thickness of the structure $(t_s) = 15\,\text{m}$ and mass density of the structure $(\rho_s) = 2400\,\text{kg/m}^3$. Fig. 3 shows a typical finite element discretization for the fluid and the structural domains. The response of the fluid–structure system subjected to ramp acceleration is

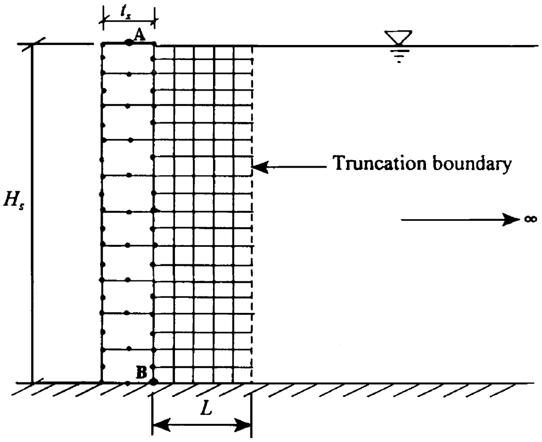

Fig. 3. Finite element mesh of a fluid–structure system.

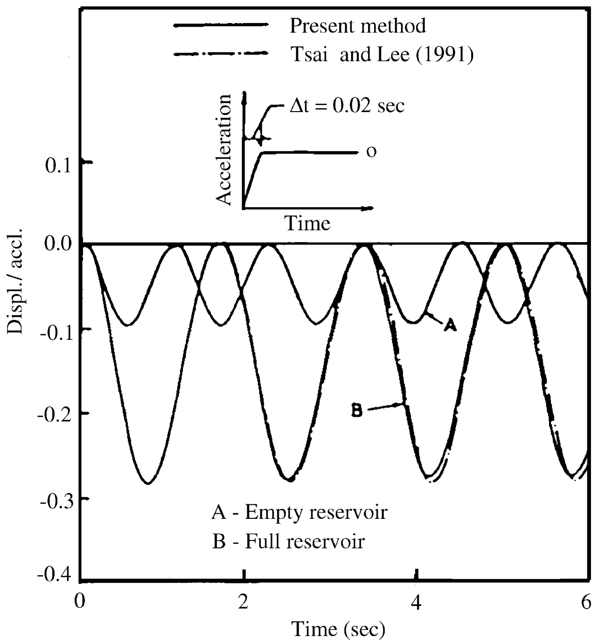

Fig. 4. Horizontal displacement at the top of the structure subjected to ramp acceleration.

studied truncating the infinite fluid domain at a distance of $L/H_f = 0.5$ and results are shown in Figs. 4 and 5. The results are compared with the method presented by Tsai and Lee [28], which validates the proposed algorithm.

Example 2. *Effect of the thickness of the structure.* To study the effect of the thickness of the structure on coupled fluid–structure system, the dimensions and material properties of the structure considered are as follows. The geometry of the coupled system is as shown in Fig. 3.

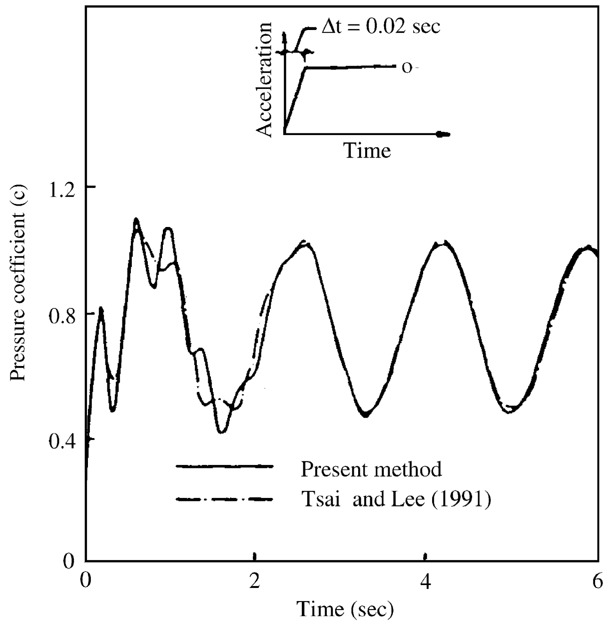

Fig. 5. Hydrodynamic pressure at the bottom of the structure subjected to ramp acceleration.

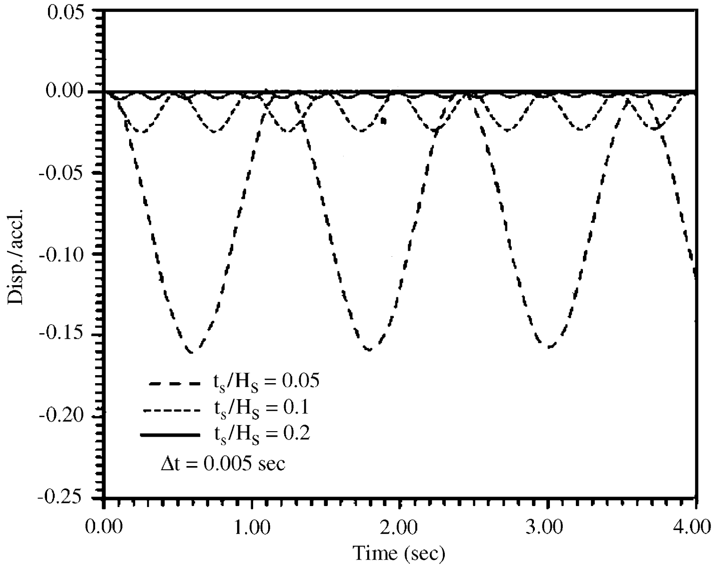

Fig. 6. Horizontal displacement at the top of the structure subjected to ramp acceleration (for different thickness of the structure).

Height of the structure (H_s) = 20.0 m; mass density of the structure (ρ_s) = 2400 kg/m^3; modulus of elasticity (E_s) = 2.85×10^7 kN/m^2, the Poisson ratio (v) = 0.20. The depth of water is 20.0 m. The infinite fluid domain is truncated at a distance of $L = H_f = 20$ m. The horizontal displacement

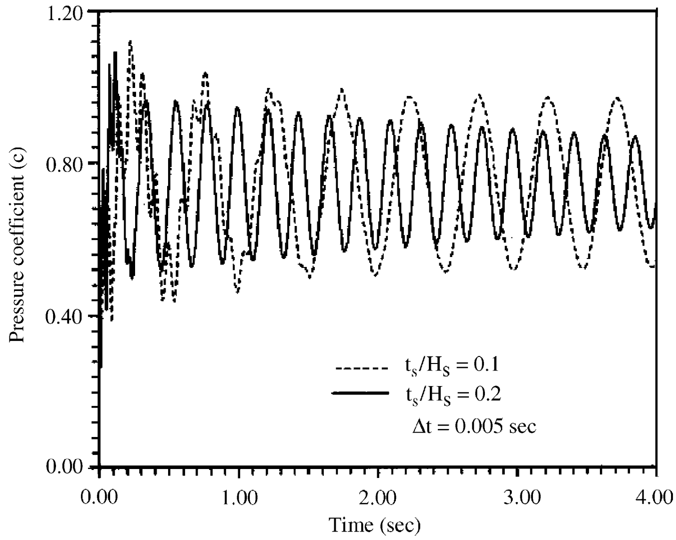

Fig. 7. Hydrodynamic pressure at the bottom of the structure subjected to ramp acceleration (for different thickness of the structure).

at the top of the structure subjected to ramp acceleration is shown in Fig. 6. The results show that the magnitude of the displacement becomes high as the thickness of the structure is reduced, as expected. The hydrodynamic pressure, developed at the bottom of the fluid–structure interface, for different structural flexibility is indicated in Fig. 7. The results, plotted in Fig. 7, depict that the magnitude of the developed hydrodynamic pressure increases with the increase of the flexibility of the structure. The same trend is observed in Fig. 9, which shows the development of the hydrodynamic pressure at the bottom of the fluid–structure interface for different thickness of the structure, when the system is subjected to the ground acceleration of El Centro earthquake (Fig. 8). The velocity vectors in the reservoir due to excitation of the coupled fluid–structure system are plotted in Figs. 10 and 11. In the plot, vertical axis represents the height of the structure and the horizontal axis represents the length of the reservoir from the fluid–structure interface. Figs. 10 and 11 clearly show that as the thickness of the structure decreases, the fluid generates a tendency to rotate more as expected physically.

Example 3. *Effect of the modulus of elasticity of the structure*. The coupled behaviour of the fluid–structure system depends on the material properties of the structure. The magnitude of the modulus of elasticity of the structure is varied by choosing different materials and the responses of the coupled system are studied. The dimensions and material properties of the fluid–structure system are similar to the one as considered in Example 2. The thickness of the structure is considered as 2.0 m ($t_s/H_s = 0.1$) in the present analysis. The results are shown in Figs. 12 and 13. The displacement at the top of the structure subjected to earthquake forces (Fig. 8) is shown in Fig. 12. The results show that the displacement increases with the decrease of elastic modulus of the structure. The hydrodynamic pressure developed at the bottom of the structure for different values of elastic modulus is shown in Fig. 13.

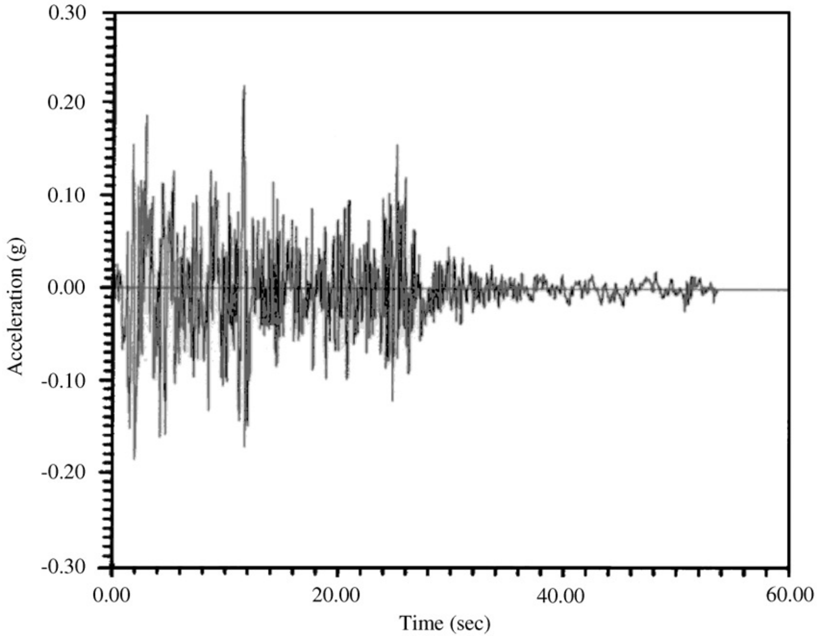

Fig. 8. Ground acceleration due to El Centro earthquake 1940 (S90W).

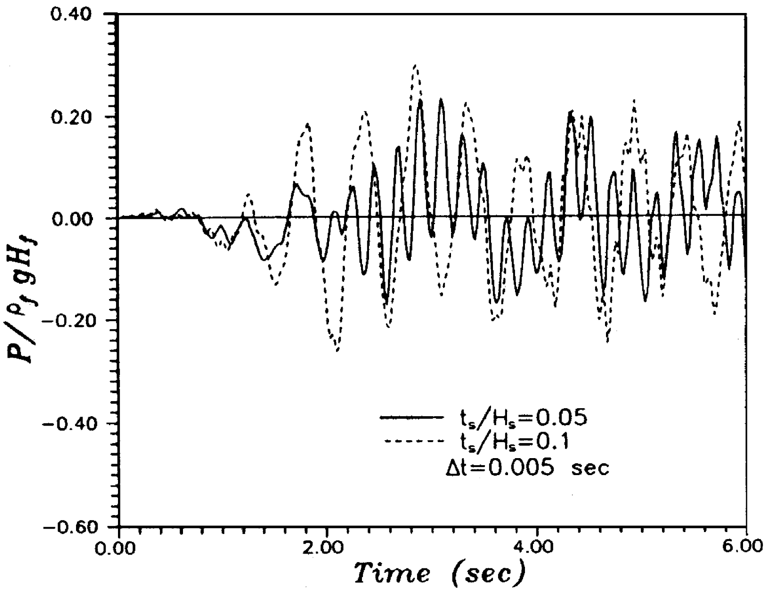

Fig. 9. Hydrodynamic pressure at the bottom of the structure subjected to El Centro earthquake (S90W).

Example 4. *Effect of water level in the dam vibration.* Fig. 14 shows the geometry and dimensions of a typical dam, which is exposed to an infinite reservoir. The mass density, the Poisson ratio and elastic modulus of the dam are $2400\,kg/m^3$, 0.15 and $3.12 \times 10^7\,kN/m^2$, respectively. The height of

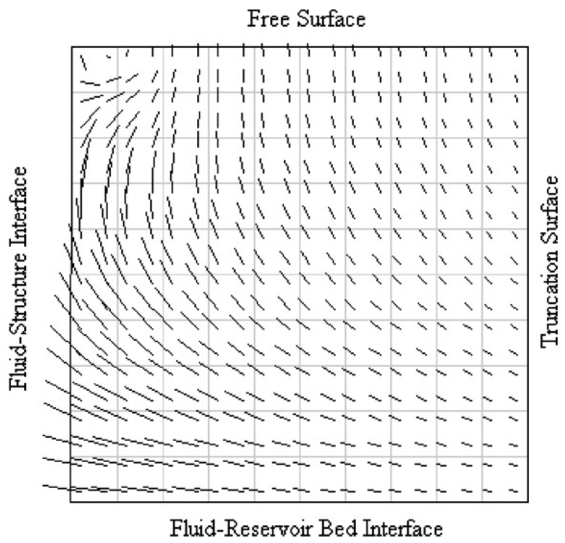

Fig. 10. Velocity vectors in the fluid domain at 2 s (thickness/height = 0.05).

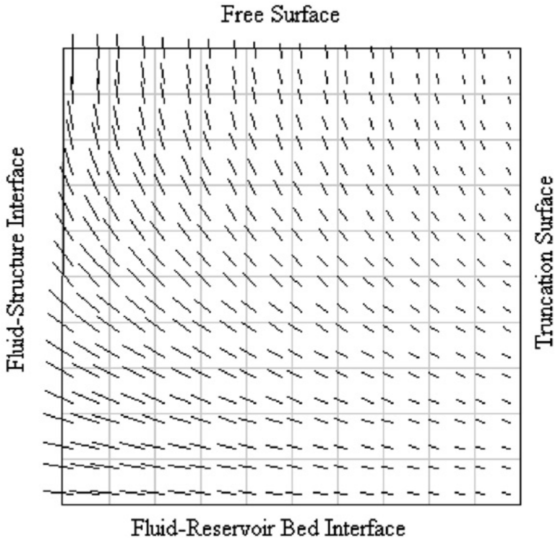

Fig. 11. Velocity vectors in the fluid domain at 2.0 s (thickness/height = 0.1).

water level (H_f) is varied to study its effect on the dam. The infinite reservoir has been truncated at a distance of half of the water depth. The dam–reservoir system is subjected to earthquake ground accelerations (Fig. 15). The hydrodynamic pressures developed at the bottom of the dam–reservoir interface for different height of water level are shown in Fig. 16. The hydrodynamic pressure at the bottom of the dam–reservoir interface increases with the increase of water level. The horizontal displacement at the crest of the dam is plotted in Fig. 17. The results show that the horizontal displacement at the crest of the dam increases with the increase of the water level in the reservoir.

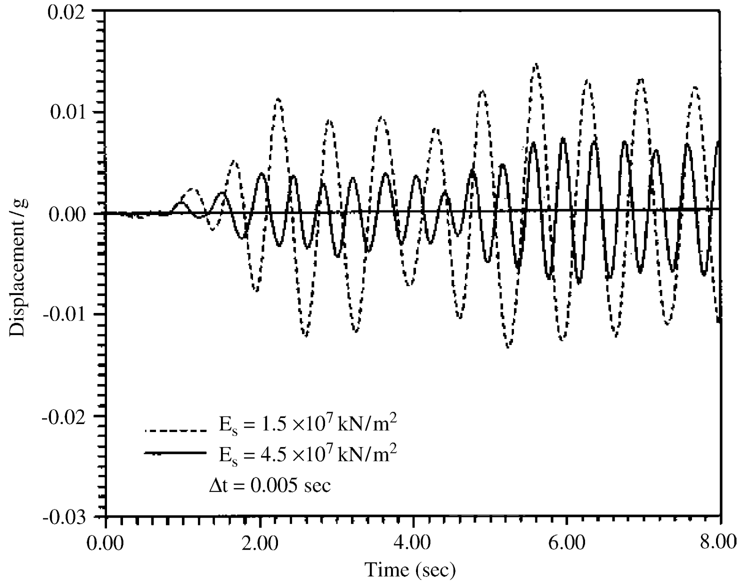

Fig. 12. Horizontal displacement at the top of the structure subjected to El Centro earthquake (S90W).

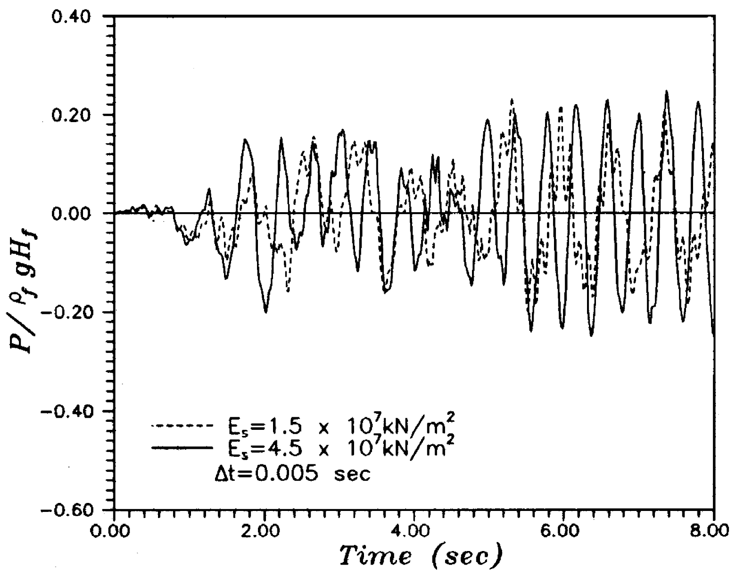

Fig. 13. Hydrodynamic pressure at the bottom of the structure subjected to El Centro earthquake (S90W).

5. Conclusion

A general time-domain procedure using finite element technique has been presented for the dynamic analysis of coupled fluid–structure systems subjected to external excitations. The two

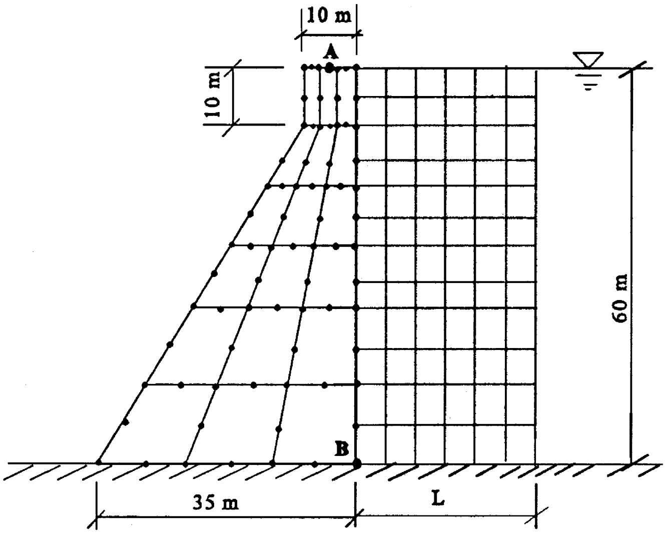

Fig. 14. Geometry and finite element discretization of a dam–reservoir system.

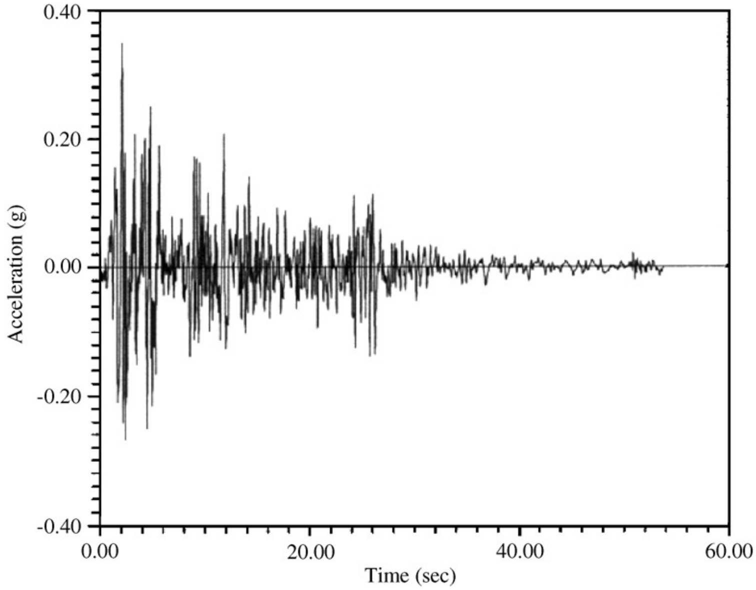

Fig. 15. Geometry acceleration due to El Centro earthquake 1940 (S00E).

different media, i.e., the fluid and the solid region are solved individually and coupled effects are obtained through the proposed iterative scheme where equilibrium conditions along the common interface are satisfied. A desired level of convergence is achieved through a few number of iterations. The major advantages of the proposed model are: (a) The resulting matrices are

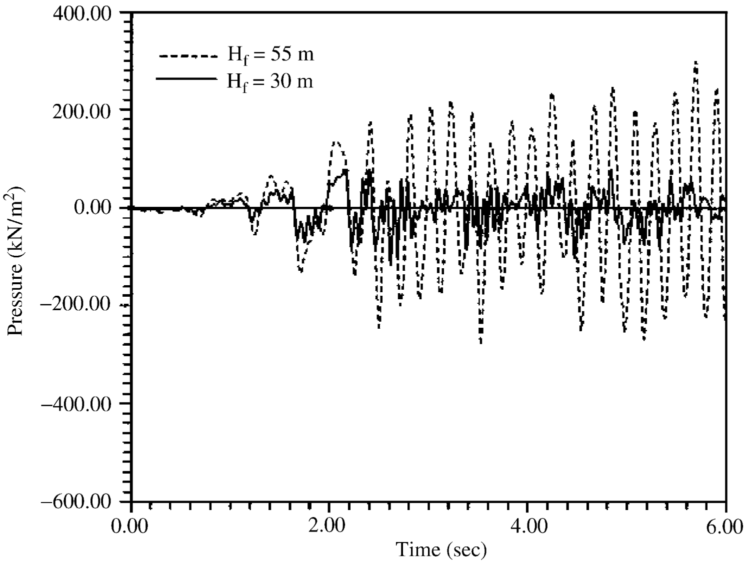

Fig. 16. Hydrodynamic pressure at the bottom of the dam for different reservoir height subjected to El Centro earthquake 1940 (S00E).

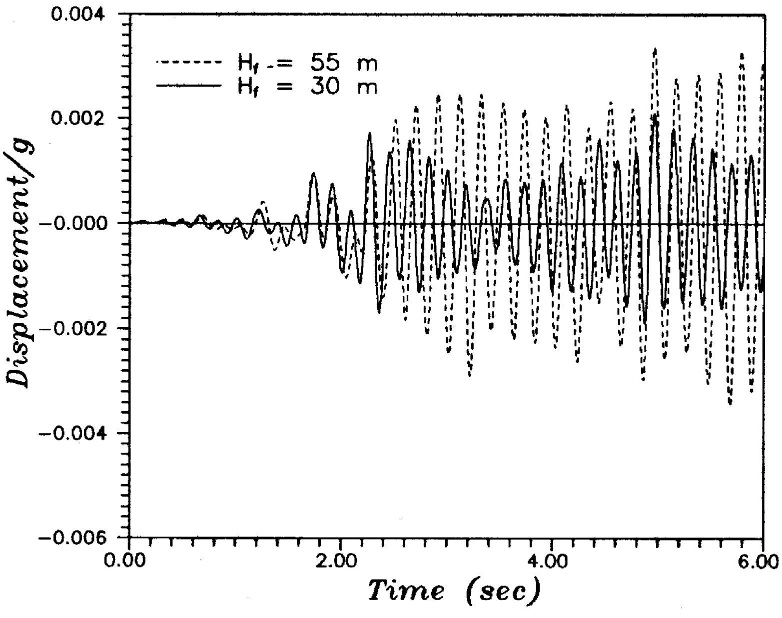

Fig. 17. Hydrodynamic displacement at the top of the dam for different reservoir height subjected to El Centro earthquake 1940 (S00E).

symmetric as the systems are dealt with separately. (b) The size of matrices required to be inverted is comparatively smaller as the two systems are solved in a decoupled manner. (c) The size of fluid domain will be considerably smaller as the infinite domain is modelled into a finite one using an efficient far-boundary condition, resulting in great computational advantages. (d) The matrices involved in the solution of the system equations are decomposed into triangular forms at the beginning of the iteration, and thereby only two forward-eliminations and back-substitutions are required at each iteration step. Thus, the time required to obtain the coupled response for a particular time instant is minimized.

The flexibility property of the structures may alter the behaviour of the fluid domain significantly. There is a common belief that in case of rigid structures, the magnitude of the hydrodynamic pressure becomes high. But this is not always true. The magnitude of the hydrodynamic pressure may increase significantly for the flexible structures as well. Moreover, if the resonance between the two systems occurs, the developed hydrodynamic pressure may increase manifold. Hence, the structures, exposed to fluid media are to be analyzed, paying due considerations to these aspects.

Appendix A. Development of truncation boundary condition

The specification of the far-boundary condition is one of the most important features in the development of reservoir model due to the fact that the hydrodynamic pressure on the structure is highly sensitive to the behaviour of the infinite region of the reservoir. In order to consider the effect of radiation damping, it is assumed that at infinitely large distance away from the structure, developed hydrodynamic pressure becomes zero. If the unbounded fluid domain is truncated at a sufficiently large distance away from the region of interest, Sommerfeld radiation boundary [5] is normally used at the truncation surface. Some recent studies [29] based on the frequency-domain analysis of two dimensional unbounded fluid reservoirs, have indicated that the Sommerfeld damper does not truly represent the effects of radiation damping, particularly when the excitation frequency is less than the second natural frequency of the reservoir. Such a range of excitation frequencies is of greater practical importance, as in the analysis of structures subjected to seismic loadings. The proposed boundary condition along the truncation surface is derived on the basis of the following assumptions (Fig. 1):

(i) The bottom of the fluid domain is horizontal and rigid.
(ii) The fluid–structure interface is vertical.
(iii) The fluid domain extends to infinity and its motion is two dimensional.

The following assumptions are made for the development of truncation boundary condition:

$$\frac{\partial p}{\partial n}(0, y, t) = -\rho_f a e^{i\omega t}, \tag{A.1}$$

$$\frac{\partial p}{\partial n}(x, 0, t) = 0 \tag{A.2}$$

$$p(x, H, t) = 0 \tag{A.3}$$

$$p(\infty, y, t) = 0 \tag{A.4}$$

The general solution of Eq. (1) satisfying Eqs. (A.1)–(A.4), hydrodynamic pressure at any point (x, y) is given by

$$p = \frac{4\rho_f aH}{\pi}\sum_{m=1}^{\infty}\frac{(-1)^{m+1}}{(2m-1)f_m}e^{(-f_m x/H)}\cos\left(\lambda_m\frac{y}{H}\right)e^{i\omega t}, \tag{A.5}$$

where

$$\lambda_m = \frac{(2m-1)\pi}{2}, \quad f_m = \sqrt{\lambda_m^2 - \Omega^2} \quad \text{and} \quad \Omega = \omega H/C. \tag{A.6}$$

Here a is the acceleration of the solid surface normal to the outwardly drawn. The partial derivative of the hydrodynamic pressure with respect to x considering Eq. (A.5) is given by

$$\frac{\partial p}{\partial x} = -\frac{4\rho_f a}{\pi}\sum_{m=1}^{\infty}\frac{(-1)^{m+1}}{2m-1}e^{(-f_m x/H)}\cos\left(\lambda_m\frac{y}{H}\right)e^{i\omega t}. \tag{A.7}$$

From Eqs. (A.5) and (A.7), the following equation is obtained:

$$\frac{\partial p}{\partial n} = \frac{\partial p}{\partial x} = -\frac{p}{H}\zeta, \tag{A.8}$$

where p is the pressure of the fluid domain and ζ is given by

$$\zeta = -\frac{\sum_{m=1}^{\infty}\frac{(-1)^{m+1}}{2m-1}e^{(-f_m x/H)}\cos\left(\lambda_m\frac{y}{H}\right)}{\sum_{m=1}^{\infty}\frac{(-1)^{m+1}}{(2m-1)f_m}e^{(-f_m x/H)}\cos\left(\lambda_m\frac{y}{H}\right)}. \tag{A.9}$$

To get the effect of unbounded fluid domain in the truncation surface, ζ is determined numerically assuming m to be a large number.

Appendix B. Nomenclature

a	acceleration of the fluid–structure interface in the normal direction
$\{B\}$	vector represents the boundary conditions of the fluid domain
$\{B_f\}$	boundary conditions at the free surface of the fluid domain
$\{B_r\}$	boundary conditions at the fluid–reservoir bed interface
$\{B_s\}$	boundary conditions at the fluid–structure interface
$\{B_t\}$	boundary conditions at the truncation surface of the fluid domain
$\{\bar{B}\}$	strain displacement matrix
C	acoustic speed of the fluid
c	pressure coefficient in the fluid domain
$[C]$	damping matrix of the structure
$[D]$	constitutive matrix of the structure
E	modulus of elasticity of the structure
$\{F_e\}$	vector for time-dependent external forces

$\{F_h\}$	vector for hydrodynamic forces
H	height of the fluid domain
$[K]$	stiffness matrix of the structure
L	length between fluid–structure interface and truncation surface
m	any number
$[M]$	mass matrix of the structure
$[N]$	shape function matrix
p	hydrodynamic pressure in the fluid domain
$\{\bar{p}\}$	vector for pressure at the element nodes
$\dot{p}$	derivative of pressure with respect to t
$\ddot{p}$	double derivative of pressure with respect to t
t	time
T	time period
U_i	displacement at ith node
$\ddot{U}_g$	ground acceleration
Δt	time step
ε	a preassigned small value
ζ	coefficients at the truncation surface in the fluid domain
ρ_f	mass density of fluid
ρ_s	mass density of structure
ω	circular frequency of vibration
v	the Poison ratio

References

[1] H.C. Chen, R.L. Taylor, Vibration analysis of fluid solid systems using a finite element displacement formulation, International Journal of Numerical Methods in Engineering 29 (1990) 683–698.

[2] G. Fenves, L.M. Vargas-Loll, Nonlinear dynamic analysis of fluid–structure systems, Journal of Engineering Mechanics, American Society of Civil Engineers 114 (1988) 219–240.

[3] L.G. Olson, K.J. Bathe, Analysis of fluid–structure interactions. A direct symmetric coupled formulation based on the fluid velocity potential, Computers & Structures 21 (1985) 21–32.

[4] O.C. Zienkiewicz, P. Bettes, Fluid–structure dynamic interaction and wave forces. An introduction to numerical treatment, International Journal of Numerical Methods in Engineering 13 (1978) 1–16.

[5] O.C. Zienkiewicz, R.E. Newton, Coupled vibration of a structure submerged in a compressible fluid, International Symposium on Finite Element Techniques, Stuttgart, Germany, 1969.

[6] W.C. Müller, Simplified analysis of linear fluid–structure interaction, International Journal of Numerical Methods in Engineering 17 (1981) 113–121.

[7] G. Sandberg, P. Göransson, Asymmetric finite element formulation for acoustic fluid–structure interaction analysis, Journal of Sound and Vibration 123 (3) (1988) 507–515.

[8] G. Sandberg, A new strategy for solving fluid–structure problems, International Journal of Numerical Methods in Engineering 38 (1995) 357–370.

[9] O.C. Zienkiewicz, D.K. Paul, E. Hinton, Cavitation in fluid–structure response (with particular reference to dams under earthquake loading), Earthquake Engineering & Structural Dynamics 11 (1983) 463–481.

[10] A. Bermúdez, R. Durán, M.A. Muschietti, R. Rodríguez, J. Solomin, Finite element vibration analysis of fluid–solid systems without spurious modes, SIAM Journal of Numerical Analysis 32 (1995) 1280–1295.

[11] D. Maity, S.K. Bhattacharyya, Response of structural system in fluid environment, in: P.M. Mujumdar, A. Joshi (Eds.), Advances in Testing, Design, and Development of Aerospace Structures, Proceedings of the Fifth National Seminar on Aerospace Structures, Indian Institute of Technology, Bombay, Allied Publishers Ltd., India, 1996, pp. 431–441.
[12] L.G. Olson, K.J. Bathe, A study of displacement-based fluid finite elements for calculating frequencies of fluid and fluid–structure systems, Nuclear Engineering and Design 76 (1983) 137–151.
[13] A.K. Chopra, P. Chakraborti, Earthquake analysis of concrete gravity dams including dam–water–foundation rock interaction, Earthquake Engineering & Structural Dynamics 9 (1981) 363–383.
[14] J.F. Hall, A.K. Chopra, Two-dimensional dynamic analysis of concrete gravity and embankment dams including hydrodynamic effects, Earthquake Engineering & Structural Dynamics 10 (1982) 305–332.
[15] M.K. Au-Yang, J.E. Galford, Fluid–structure interaction—a survey with emphasis on its application to nuclear steam system design, Nuclear Engineering and Design 70 (1982) 387–399.
[16] I. Antoniadis, A. Kanarachos, Decoupling procedures for fluid–structure interaction problems, Computer Methods in Applied Mechanics and Engineering 70 (1988) 1–25.
[17] R.K. Singh, T. Kant, A. Kakodkar, Coupled shell–fluid interaction problems with degenerate shell and three-dimensional fluid elements, Computers & Structures 38 (1991) 515–528.
[18] D. Maity, S.K. Bhattacharyya, Time domain analysis of infinite reservoir by finite element method using a novel far-boundary condition, International Journal of Finite Elements in Analysis and Design 32 (1999) 85–96.
[19] S.K. Sharan, Finite element analysis of unbounded and incompressible fluid domains, International Journal of Numerical Methods in Engineering 21 (1985) 1659–1669.
[20] S.K. Sharan, Time-domain analysis of infinite fluid vibration, International Journal of Numerical Methods in Engineering 24 (1987) 945–958.
[21] S.S. Saini, P. Bettess, O.C. Zienkiewicz, Coupled hydrodynamic response of concrete gravity dams using finite and infinite elements, Earthquake Engineering & Structural Dynamics 6 (1978) 363–374.
[22] Y.G. Hanna, J.L. Humar, Boundary element analysis of fluid domain, Journal of Engineering Mechanics, American Society of Civil Engineers 108 (1982) 436–450.
[23] C.S. Tsai, G.C. Lee, Arch dam–fluid interactions: by fem-bem and substructure concept, International Journal of Numerical Methods in Engineering 24 (1987) 2367–2388.
[24] A.T. Chwang, Hydrodynamic pressure on sloping dams during earthquakes, Part 2: exact theory, Journal of Fluid Mechanics 87 (1978) 343–348.
[25] H.M. Westergaard, Water pressure on dams during earthquakes, Transactions of the American Society of Civil Engineers 98 (1933) 418–472.
[26] R. Yang, C.S. Tsai, G.C. Lee, Explicit time-domain transmitting boundary for dam–reservoir interaction analysis, International Journal of Numerical Methods in Engineering 36 (1969) 1789–1804.
[27] R.W. Clough, R.M. Stephen, J.S.-H. Kuo, Dynamic response analysis of Techi dam, Report No. EERC 82-09, Earthquake Engineering Research Center, University of California, Berkeley, CA, 1982.
[28] C.S. Tsai, G.C. Lee, Time-domain analysis of dam–reservoir system II: substructure method, Journal of Engineering Mechanics, American Society of Civil Engineers 117 (1991) 2007–2026.
[29] X. Li, O. Romo, M.P. Avilés, Finite element analysis of dam–reservoir systems using an exact far-boundary condition, Computers & Structures 60 (1996) 751–762.